Dynamics of the COVID-19 Comparison between the Theoretical Predictions and the Real Data, and Predictions about Returning to Normal Life

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1.Abstract

A new coronavirus disease, called COVID-19, appeared in the Chinese region of Wuhan at the end of last year; since then the virus spread to other countries, including most of Europe. We propose a differential equation governing the evolution of the COVID-19. This dynamic equation also describes the evolution of the number of infected people for 13 common respiratory viruses (including the SARS-CoV-2). We validate our theoretical predictions with experimental data for Italy, Belgium and Luxembourg, and compare them with the predictions of the logistic model. We find that our predictions are in good agreement with the real world since the beginning of the appearance of the COVID-19; this is not the case for the logistic model that only applies to the first days. The second part of the work is devoted to modelling the descending phase, i.e. the decrease of the number of people tested positive for COVID-19. Also in this case, we propose a new set of dynamic differential equations that we solved numerically. We use our differential equations parametrised with experimental data to make several predictions, such as the date when Italy, Belgium, and Luxembourg will reach a peak number of SARS-CoV- 2 infected people. The descending curves provide valuable information such as the duration of the COVID-19 epidemic in a given Country and therefore when it will be possible to return to normal life.

We find ourselves in a global pandemic, referred to as COVID-19. There is much research underway on all aspects of the pandemic, including to slow its spread, improve diagnostic tests, develop a vaccine, and mathematical models able to foresee the dynamic of this pandemic. In this paper, we develop a mathematical model for the spread of the coronavirus disease 2019. By means of a very simple mathematical model, we study the particular case of Italy, Belgium, and Luxembourg and we provide the dynamic of the descending phase, i.e. the evolution of the decrease number of people tested positive to the COVID-19. The predictions about the descending phase provide valuable information about the duration of the COVID-19 in a given Country, especially when it will be possible to return to normal life. The theoretical predictions are in excellent agreement with the experimental data.

3. Introduction

Viral infections usually affect the upper or lower respiratory tract. Although respiratory infections can be classified according to the causative agent (e.g. the flu), they are mostly clinically classified according to the type of syndrome (e.g., common cold, bronchiolitis, laryngo-tracheo-bronchitis acute, pneumonia). Although pathogens typically cause characteristic clinical manifestations (e.g., rhinovirus causes the common cold, respiratory syncytial virus [RSV] usually causes bronchiolitis), they can all cause many of the most common respiratory syndromes.

The severity of viral respiratory disease is highly variable; serious illness is more frequent in elderly patients and young children. Morbidity can either directly result from the infecting agent, or may be indirect. The latter case can be due to the exacerbation of an underlying cardiopulmonary disease, or a bacterial superinfection of the lung, paranasal sinuses, or middle ear. The main motivation of this work is to verify, by making theoretical predictions, that political decisions are truly effective to minimise the number of infected people in order to (i) not overload local health services (such as hospitals), and to (ii) gain time to allow research institutes to deliver vaccines or the anti-virals.

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(Table 1 and Table 3) respectively provide the experimental data for Italy [1] and for Belgium [2, 3]. They show the number of active people (i.e., people currently infected by SARS-CoV-2), the recovered people, and deaths for COVID-19.

We start our theoretical analysis by introducing the definition of the basic re-production number of an infection R_0, defined as the number of infected people derived from a first case in a population where all the others are susceptible. So, it is not possible to modify R_0, in any case, but it is possible to get a different effective R[1]. This parameter is strictly linked to the replication time of a virus, indicated with μ_i, defined as the time interval after which the number of infected people has increased by R_0 times. (Figure 1) schematically represents the diffusion dynamics of the virus. By indicating with N the number of infected people, after n steps we get[2]:

$$N = R_0^n$$
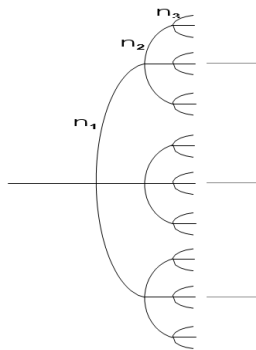
Figure 1: Schematic dynamics of respiratory virus in the absence of the lockdown measures in this graphics, for illustrative purpose only, we set $R_0 = 3$. However, for SARS-CoV-2, the value of R is 2 even at the beginning of the outbreak in China and Italy. After a period of time $\mu1$, an infected individual can infect R_0 other individuals. In turn, after a period $\mu2$, each of these newly infected individuals can infect other R_0 people, and so on. After n steps the elapsed time is $t = \sum_{i=1}^{n} \mu_i$

Table 1: *Situation in Italy on 15 May 2020*. Columns report the number of active people (currently infected by SARS-CoV-2), the number of recovered people, and the number of deceased people.

Date	Active	Recovered	Deceased	Total cases
25-Feb	322	1	10	333
26-Feb	400	3	12	415
27-Feb	650	45	18	713
28-Feb	888	46	21	955
29-Feb	1049	50	29	1128
1-Mar	1577	83	34	1694
2-Mar	1835	149	52	2036
3-Mar	2263	160	79	2502
4-Mar	2706	276	107	3089
5-Mar	3296	414	148	3858
6-Mar	3916	523	197	4636
7-Mar	5061	589	233	5883
8-Mar	7375	622	366	8363
9-Mar	9172	724	463	10359
10-Mar	10149	1004	631	11784
11-Mar	10590	1045	827	12462
12-Mar	12839	1258	1016	15113
13-Mar	14955	1439	1266	17660
14-Mar	17750	1966	1441	21157
15-Mar	20603	2335	1809	24747
16-Mar	23073	2749	2158	27980
17-Mar	26062	2941	2503	31506
18-Mar	28710	4025	2978	35713
19-Mar	33190	4440	3405	41035
20-Mar	37860	5129	4032	47021
21-Mar	42681	6072	4825	53578
22-Mar	46638	7024	5475	59137
23-Mar	50418	7432	6077	63927
24-Mar	54030	8326	6820	69176
25-Mar	57511	9362	7503	74376

Date	Active	Recovered	Deceased	Total cases
26-Mar	62013	10361	8165	80539
27-Mar	66414	10950	9134	86498
28-Mar	70065	12384	10023	92472
29-Mar	73880	13030	10779	97689
30-Mar	75528	14620	11591	101739
31-Mar	77635	15729	12428	105792
1-Apr	80572	16847	13155	110574
2-Apr	83049	18278	13915	115242
3-Apr	85388	19758	14681	119827
4-Apr	88274	20996	15362	124632
5-Apr	91246	21815	15887	128948
6-Apr	93187	22837	16523	132547
7-Apr	94067	24392	17127	135586
8-Apr	95362	26491	17669	139422
9-Apr	96877	28470	18279	143626
10-Apr	98273	30455	18849	147577
11-Apr	102253	34211	19899	152271
12-Apr	100269	32534	19468	156363
13-Apr	103616	35435	20465	159516
14-Apr	104291	37130	21067	162488
15-Apr	105418	38092	21645	165155

Table 2: Situation in Italy on 15 May 2020. Columns report the number of active people (currently infected by SARS-CoV-2), the number of recovered people, and the number of deceased people.

Date	Active	Recovered	Deceased	Total cases
16-Apr	106607	40164	22170	168941
17-Apr	106962	42727	22745	172434
18-Apr	107771	44927	23227	175925

Date	Active	Recovered	Deceased	Total cases
19-Apr	108257	47055	23660	178972
20-Apr	108237	48877	24114	181228
21-Apr	107709	51600	24648	183957
22-Apr	107699	54543	25085	187327
23-Apr	106848	57576	25549	189973
24-Apr	106527	60498	25969	192994
25-Apr	105847	63120	26348	195351
26-Apr	106103	64928	26644	197675
27-Apr	105813	66624	26977	199414
28-Apr	105205	68941	27359	201505
29-Apr	104657	71252	27682	203591
30-Apr	101551	75945	27967	205463
1-May	100946	78249	28236	207428
2-May	100704	79914	28710	209328
3-May	100179	81654	28884	210717
4-May	99980	82879	29079	211938
5-May	98467	85231	29315	213013
6-May	91528	93245	29684	214457
7-May	89624	96276	29958	215858
8-May	87961	99023	30201	217185
9-May	84842	103031	30395	218268
10-May	83324	105186	30560	219070
11-May	82488	106587	30739	219814
12-May	81266	109039	30911	221216
13-May	78457	112541	31106	222104
14-May	76440	115288	31368	223096
15-May	72070	120205	31610	223885
16-May	70187	122810	31763	224760
17-May	68351	125176	31908	225435
18-May	66553	127326	32007	225886
19-May	65129	129401	32169	226699

20-May	62752	132282	32330	227364	3-May	100179	81654	28884	210717
21-May	60960	134560	32486	228006	4-May	99980	82879	29079	211938
22-May	59322	136720	32616	229858	5-May	98467	85231	29315	213013
23-May	57752	138840	32735	229327	6-May	91528	93245	29684	214457
24-May	56594	140479	32785	229858	7-May	89624	96276	29958	215858
25-May	55300	141981	32877	230158	8-May	87961	99023	30201	217185
26-May	52942	144981	32955	230555	9-May	84842	103031	30395	218268
27-May	50966	147101	33072	231139	10-May	83324	105186	30560	219070
28-May	47986	150604	33142	231732	11-May	82488	106587	30739	219814
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2-May	100704	79914	28710	209328					

[1]More rigourously, in epidemiology, the basic reproduction number of an infection, R_0, is the expected number of cases directly generated by one case in a population where all individuals are susceptible to infection in absence of any deliberate intervention in disease transmission (see, for example, [4]).

Of course, after n steps, the elapsed time is $t = \sum_{i=1}^{n} \mu_i$ and, if there are M outbreaks of infectious viruses, Eq. (1) can be cast into the form[3]
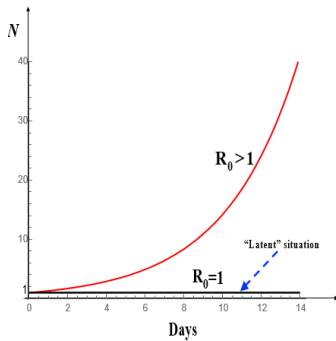
$$N = MR_0^{t/\mu} \tag{2}$$

with $t = 1/n \sum_{i=1}^{n} \mu_i$ Note that the two parameters R_0 and μ are not independent (see, for example, [7-9])[4]. It is more convenient to work in the Euler base e rather than in base R_0; in the Euler base Eq. (2) provides the law of growth of a Malthusian population [5].

$$N = M \exp(t/\tau) \text{ where } \tau = \frac{\mu}{\log(R_0)} \tag{3}$$

In literature, τ is referred to as the characteristic time of the exponential trend. So, in the absence of containment measures the number of infected people follows the exponential law (3). Let us now analyse Eq. (3) in more dept. We have three possible scenarios:

1. $R_0 > 1$ (as is the current world's situation). For Italy, for example, before the adoption of (severe) containment measures, the value of τ was about $\tau \sim 3.8$ days (and $\mu \sim 2.6$ days). In this case the number of the infected people increases exponentially.

2. $R_0 = 1$ If the infection-capacity of the virus is of the type one-to-one (i.e., a person infected by SARS-CoV-2 can in turn infects only another person), we get the stationary situation corresponding to N = 1. This situation is referred to as the latent situation i.e., the virus is still present but does not spread. In this limit case, the SARS-CoV-2 is substantially ineffective. Scenarios (1) and (2) are illustrated in (Figure 2).
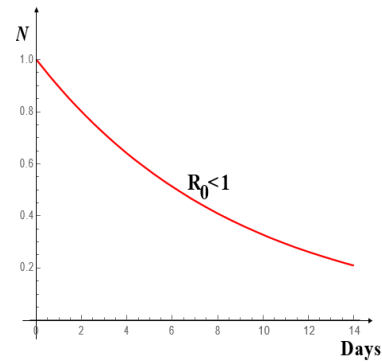

Figure 2: Situation before the lockdown measures. Number of infected people corresponding to the exponential law. The red line represents the case $R_0 > 1$, such as the situation before the adoption of lockdown measures. The black line corresponds to the case $R_0 = 1$, the latent situation in which the virus is substantially ineffective.

3. $0 < R_0 < 1$. We may also imagine that the capacity of infection of SARS-CoV-2 is less than 1. This means that the virus is no longer able to be spread (e.g., thanks to protective measures, or to the production of vaccines and anti-virals, or because people who overcame the disease became immune. In this case, the value of τ is negative and the number of infected people decreases ever time. That is, the infection eventually disappears. The rate of decrease of the number of infected people depends on the value of τ. This scenario is depicted in (Figure 3).

Figure 3: Number of infected people corresponding to the exponential law. The red line represents the case $R_0 < 1$. In this situation the number of infected people decreases exponentially and the virus disappears after a few weeks.

4. Comparison with the Real Data for COVID-19 before the Lockdown Measures

From a mathematical point of view, we would like to have R0 = 1 (or, better, R0 < 1), in Eq. (3) instead of R0 > 1. In practical terms, this means reducing the frequency of all involuntary It is understood that the main objective of the lockdown measures established by most European governments and health organisations is to reduce the ability of a virus to spread. contacts with a large number of

[2]In this Section we shall follow the definitions and the expressions reported in standard books or thesis dissertation such as, for example, [5, 6].

[3]Actually, Eq. (2) applies only if the M outbreaks of the virus are exactly at the same conditions. In general, the correct expression reads $N = \sum_{i=0}^{M} R_0^{t/\tilde{\mu}_i}$ with $\tilde{\mu}_i$ indicating the replication time of the virus for the i-th outbreak.

[4]In ref. [7], the doubling time is used to calculate R_0, by means of the equation $R_0 = 1 + (\gamma + \rho) \log(2)/\mu$ where γ is the duration of the incubation period, ρ is the duration of the symptomatic period, and μ is the doubling time (see [7]). In this respect, we would also like to mention another excellent work recently produced by G. Steinbrecher [9] (Figure 2 and 3).

People, reducing unnecessary movements to avoid enc-ounters, and to prolong the closure of schools. Although these measures cannot prevent the spread of the infection in the long term, they can reduce the number of new infections daily. This has the benefit of leaving room for seriously-ill patients by avoiding to overload the healthcare system. We can easily realise what are the consequences if the lockdown measures are not set up. To make a comparison between the theoretical predictions and the experimental data in absence of lockdown measures, we have to consider the correct reference period. More specifically, we saw that the number of positive cases grows in the course of time by following the law (3). Hence, at the reference time t0, the number of people infected by the virus is

$$N_0 = M\exp(t_0/\tau) \qquad (4)$$

After a period of time, say t, Eq. (3) reads

$$N = M \exp(t/\tau) \qquad (5)$$

Hence,

$$N = N_0 M\exp(t - t_0/\tau) \qquad (6)$$

Eq. (6) is the equation that we use for comparing the mathematical predictions with experimental data during the initial phase where the spread of SARS- CoV-2, causing the COVID-19, follows the exponential law, and (t-t0) is our reference period. For the case of COVID-19 we get (see, for example, [5, 6])

All infectious outbreaks are exactly at the same conditions. So, Eq. (2) applies;

$R_0 = 2$;

All the μ_i are equal with each other: $\mu_i = const = \mu$ (see also [5, 7]).

In this case, μ is referred to as the doubling time. So, the doubling time is the amount of time it takes for a given quantity to double in size or value at a constant growth rate [8]. If we do not apply the locking measures, the evolution in the course of time of the number of infected people is best approximated by an exponential curve with R = 2, even though we have to stress that R_0 is only associated with the beginning of the epidemic and, with certain approximations, with the early stages, but not beyond. (Figure 4 and Figure 5) respectively show the comparison between the theoretical predictions and the experimental data for Italy and Belgium before the lockdown measures. We get $\tau \simeq 3.8$ days and $\mu \simeq 2.6$ days for Italy, and $\tau \simeq 5.2$ days and $\mu \simeq 3.7$ days for Belgium. We conclude this Introduction by mentioning that there are several methods currently proposed in Literature to derive by mathematical models, the value of R_0. For example, in ref. [9], we have a short numerical code, written in R-programming language for statistical computing and graphics, able to compute the estimated R_0 values for the following 17 infectious diseases: Chickenpox (varicella) (Transmission: Aerosol), Common cold (Transmission: Respiratory Droplets), COVID-19 (Transmission: Respiratory Droplets), Diphtheria (Transmission: Saliva), Ebola - 2014 Ebola outbreak (Transmission:: Body fluids, HIV/AIDS (Transmission: Body fluids), Influenza - 1918 pandemic strain (Transmission:: Respiratory Droplets), Influenza - 2009 pandemic strain (Transmission: Respiratory Droplets, Influenza - seasonal strains (Transmission: Respiratory Droplets), Measles (Transmission: Aerosol), MERS (Transmission: Respiratory Droplets), Mumps (Transmission: Respiratory Droplets), Pertussis (Transmission: Respiratory Droplets), Polio (Transmission: Fecal oral route), Rubella (Transmission: Respiratory Droplets), SARS (Transmission: Respiratory Droplets), Smallpox (Transmission: Respiratory Droplets). However, this task is particularly problematic if there are intermediate vectors between hosts, such as malaria.

This manuscript is organised as follows. Section (2) determines the dynamic differential equation for the COVID-19; Section (2.3) compares the theoretical predictions and experimental data for Italy and Belgium. The differential equations providing the evolution of the decrease of the number of people tested positive for COVID-19 can be found in Section (3); Section (4) concludes. The comparison between the theoretical predictions of our model and experimental data for Luxembourg, as well as the solution of the differential equations in the descending phase for Luxembourg are reported in Appendix. We may object that we are dealing data from countries which have passed the peak of infection, such as South Korea, Iceland or Austria etc. The situation in other Countries, which may have adopted other political decisions about the application of the lockdown measures, may be subject of future works. However, we would like to mention that several authors are currently applying our model to other Countries. In this regard, we have received their pre-prints such as the work cited in Ref. [11]. More specifically, we have received a message where our model has been used, with success, to analyse data from UK, USA, NY City, Spain, and Mumbai City. We stress the fact that this manuscript deals with the spread of SARS-CoV-2 until May 16, 2020, as the objective of this work is to study

the effect of the strict lockdown measures. After May 2020, these measures have been modified by the various Governments, which have decided to introduce less heavy and much less restrictive lockdown measures[5].
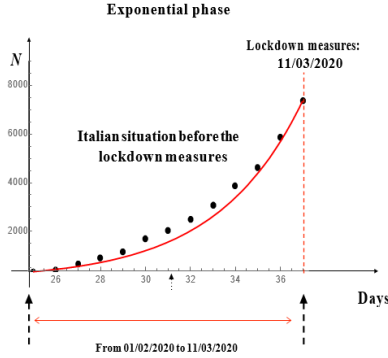
Figure 4: Number of infected people in Italy on the 10th of March 2020 (before the adoption of lockdown measures). The blue line corresponds to the theoretical predictions and the black dots correspond to experimental data. The values of the parameters τ_{IT} and μ_{IT} are $\tau_{IT} \simeq 3.8$ days and $\mu_{IT} \simeq 2.6$ days, respectively.
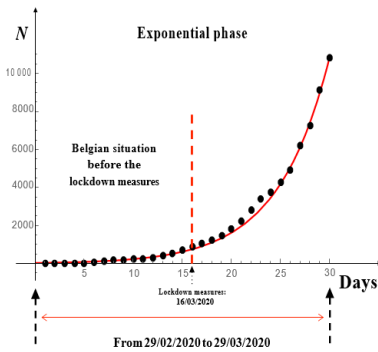
Figure 5: Exponential phase in Belgium. The lockdown measures have been adopted on the 16the of March 2020 (however, initially not so strict as in Italy). The red line corresponds to the theoretical predictions and the black dots correspond to experimental data. The values of the parameters τ_{BE} and μ_{BE} are $\tau_{BE} \simeq 5.3$ days and $\mu_{BE} \simeq 3.7$ days, respectively.

5. Modelling the COVID-19 - Virus' growth

The objective of this section is to determine the coefficients of the evolutionary differential equation for the COVID-19 (see the forthcoming Eq. (13)). We also determine the generic analytical expression for the time-dependent number of infected people through fitting techniques validated by the χ^2 tests. This expression is proposed after having previously analysed 12 respiratory infectious diseases caused by viruses [10], in

addition of being solution to the Richard's differential equation.

5.1 General Background

Letting N represent population size and t represent time, the Logistic model model is formalised by the differential equation below:

$$\frac{dN}{dt} = \alpha N \left(1 - \frac{N}{K}\right) \tag{7}$$

where $\alpha > 0$ defines the grow rate and $K > 0$ is the carrying capacity. In this equation, the early, unimpeded growth rate is modelled by the first term $+\alpha N$. The value of the rate α represents the proportional increase of the population N in one unit of time. Later, if the system is closed (i.e. the system is isolated and, hence, not in contact with a reservoir allowing the system to exchange individuals), as the population grows the modulus of the second term, $\alpha N^2/K$, becomes almost as large as the first, until to saturating the exponential growth. This antagonistic effect is called the bottleneck, and is modelled by the value of the parameter K. The competition diminishes the combined growth rate, until the value of N ceases to grow (this is called maturity of the population). The solution of Eq. (7) is

$$N(t) = \frac{K}{1 + B\exp(-t/T)} \tag{8}$$

where $B > 0$ is a constant related to the value of N (0). It is more convenient to rewrite Eq. (8) in terms of the initial Logistic time t_{0L} defined as

$$t_{0L} = \tau \log B \tag{9}$$

So, Eq. (8) may be cast into the form

$$N(t) = \frac{K}{1 + \exp(-(t - t_{0L}/\tau))} \tag{10}$$

where τ is linked to the steepness of the curve. Since the environmental condi- tions influence the carrying capacity, as a consequence it can be time-varying, with $K(t) > 0$, leading to the following mathematical model (see, for example, [12]):

$$\frac{dN}{dt} = \alpha N \left(1 - \frac{N}{K(t)}\right) \tag{11}$$

occurs. The phenomenological logistic function is used to model the evolution of the COVID-19 pandemic in different Countries. The logistic model is mainly used in More generally, the growth modelling is well described by

[5]The study of the dynamics of COVID-19 when the population is subjected to less restrictive measures is beyond the scope of this work and it will be subject of future studies.

Richards' differential equation (RDE) [13]

$$\frac{dN}{dt} = \alpha N \left(1 - \left(\frac{N}{K(t)}\right)^{v}\right) \qquad (12)$$

where $v > 0$ affects near which asymptote maximum growth epidemiology and provides insights into the transmission dynamics of the virus. We note, however, to evaluate the dynamics of transmission of SARS-CoV-2, more refined models are needed, which take into account specific measures adopted in each Country [14]. So, let us suppose that the Government decides to adopt the lockdown measures. After the application of the lockdown measures the equation may be revised to be

$$\frac{dN}{dt} = \alpha N \left(1 - \left(\frac{N}{K(t)}\right)^{v}\right) - c(t)N \qquad (13)$$

where c(t) takes into account the degree of effectiveness of the lockdown measures.

5.2. Determination of the Carrying Capacity and the Lockdown Coefficient for the COVID-19

According to ref. [15][6] Respiratory viruses remain quiet for months, inactive but viable, within living cells. Then suddenly they activate and become virulent as they say, the infectious capacity grows to a maximum, after which it decreases. The time duration is about of 2 or 3 months. So we can expect that the epidemic will soon die out in Italy too. So, there is no valid reason to think that this coronavirus behaves differently from others [15]. The present work starts from the following hypothesis: the SARS-CoV-2 behaves like the other viruses that cause respiratory diseases. As a consequence, for the COVID-19 case, functions K(t) and c(t) are determined by performing several fittings on the growth rate-trends of infection capacity of the viruses that mainly affect the respiratory system. More specifically, we considered the following 13 different diseases caused by 12 different viruses: Whooping Cough (Pertussis), Swine Flu (H1N1), Bird Flu (Avian Flu H5N1), Enterovirus, Flu in Children, Flu in Adults, Bacterial Pneumonia, Viral Pneumonia, Bronchitis, Common Cold (Head Cold), Severe acute respiratory syndrome (SARS), and MERS (Middle East Respiratory Syndrome). In all the examined cases, we took into account the fact that the therapy-induced death rate is greater than

the baseline proliferation rate, then there is the eradication of the disease. In other words, for the above-mentioned cases the function

c(t) in Eq. (13) represents the therapy-induced death rate [16, 24]. Of course, this is an oversimplified model of both the growth and the therapy (e.g., it does not take into account the phenomenon of clonal resistance). We empirically noticed (according to the $\chi 2$ test) that all these viruses have in common the same growth rate-trend of infected people (of course, each of these behaviors have their own growth rate parameters). In particular, we get that the trends of the total number of infected people by respiratory viruses (indicated with N), subject to the therapy-induced death rate, versus time satisfy the following

O.D.E. [10]

$$\frac{dN}{d\hat{t}} = \alpha N \left(1 - \frac{N}{K_N}\right) - \left(\frac{\alpha \hat{t}^2 - 1}{\hat{t}}\right) N \text{ with } \hat{t} > 1/\alpha^{1/2} \qquad (14)$$

where we have introduced the dimensionless time $\hat{t} \equiv t/t0$. The coefficient

$$c(t) \equiv \left(\frac{\alpha \hat{t}^2 - 1}{\hat{t}}\right) \quad \text{with} \quad \hat{t} > 1/\alpha^{1/2} \qquad (15)$$

is referred to as the average therapy-induced death rate. In our case the term c(t)N in the dynamic equation represents the lockdown measures. The lockdown is mainly based on the isolation of the susceptible individuals, eventually with the removal of infected people by hospitalisation[7]. In the idealised case,

for $\alpha \hat{t}^2 > 1$, c(t) may be modelled as a linear function of $\hat{t}$, by getting

$$c(\hat{t}) = \alpha \hat{t} \qquad (16)$$

As for the epidemiological explanations relating to the various modelling of c(t) (constant, linear in time etc.), we refer the reader to the well-known and extensive literature on the subject (see, for example, Ref [25] or to the references cited in [26]). Here, we limit ourselves to give a very intuitive explanation on the physical meaning of this contribution Immediately after the lockdown measures have been adopted, i.e. during the very first initial

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[6]Prof. Roberto Ronchetti is currently working at the Pediatric Clinic of La Sapienza of the University of Rome, at the Policlinico Umberto I and at the S. Andrea 24 March Hospital where he helped to found, dealt with childhood respiratory diseases, and studied bronchiolitis in particular. In these days he has studied (with his collaborators) the data available on SARS-CoV-2 in China, in South Korea and now in Italy.

[7]It is worth mentioning that initially England did not adopt any lockdown measures believing that the British system be a closed system. Basically, it was believed that the system be governed by a simply logistic equation with a carrying capacity constant or decreasing in time. However, contrary to the expectations, in England the carrying capacity did not decrease in time. This induced the British government to adopt the lockdown measures.

phase, we expect that $c(\hat{t},)$ is practically constant in time, as these measures have not yet been able to act effectively. However, after a short period of time, the positive effects of the lockdown measures become increasingly efficient and it is intuitive to expect a linear growth of $c(\hat{t})$ in time. More specifically, we expect that, after a short transition period the coefficient $c(\hat{t})$ starts to grow linearly in time. Successively, at the leading order, $c(\hat{t},)$ will be equal in magnitude to the coefficient of the linear term (in order to balance the growth rate induced by the linear term). This because the lockdown measures will be able to satuarte the exponential growth. Briefly, we expect that the O.D.E. governing the dynamics of the SARS-CoV-2 is of the form (14) where $c(\hat{t}) = \alpha\hat{t}$, for $\hat{t}, > 1/\alpha^{1/2}$. Indeed, for values of time $\hat{t} \simeq 1$, the lockdown term in Eq. (14) is able to balance the exponential grow, which is in agreement with our intuitive expectations.

From Eq. (14) we get that the time derivative of N vanishes for N = Ns, with Ns satisfying the equation

$$\frac{N_s}{K_N} = \frac{a\hat{t} - a\hat{t}^2 + 1}{a\hat{t}} > 0 \tag{17}$$

By taking into account the inequality reported in Eq. (15), we get that the O.D.E. (14) is valid in the range

$$\frac{1}{\alpha^{1/2}} < \hat{t} < \frac{(1+\alpha/4)^{1/2}}{\alpha^{1/2}} + \frac{1}{2} \tag{18}$$

Parameters K_N and α depend on the virus in question and on the external conditions (e.g. in our case, the lockdown measures) to which the population is subject. In Eq. (14), the term $-N^2 K_N$ is the term that tends to saturate the exponential growth. K_N is constant (or decreases) in the course of time since the non-linear contribution becomes more and more important until saturating the exponential growth. In our model, the carrying capacity is kept constant. For large values of the carrying capacity, the solutions of Eq. (14) reach the plateau at the time $\hat{t}_{Max}$ given by the expression

$$\hat{t}_{Max} \equiv \frac{t_{Max}}{t_0} = \frac{1}{\alpha^{1/2}} + \frac{1}{2} \tag{19}$$

Notice that α is linked to μ. Indeed, as shown in Section 1, during the exponential period the COVID-19 grows according to the law (see Eq. (6)):

$$\frac{dN}{d\hat{t}} \simeq \hat{\tau}^{-1} N \quad \text{where } \hat{\tau} \equiv \frac{\hat{\tau}}{t_0} \tag{20}$$

Hence, we get

$$\alpha \simeq \frac{1}{\hat{\tau}} = \frac{\log(R_0)}{2\hat{\mu}} \quad \text{where } \hat{\mu} \equiv \frac{\mu}{t_0} \tag{21}$$

We conclude this Section by mentioning that we may easily check (numerically) that, for systems having a large carrying capacity, the solution of Eq. (14) is well fitted by the expression

$$N \simeq At\exp(-(t-t_0)^2/\sigma) \text{ with } \sigma = 2t_0^2/\alpha \tag{22}$$

The values of parameters A, t_0 and σ depend on the virus in question. It is convenient to re-write Eq. (20) in dimensionless form

$$N \simeq \hat{A}\hat{t} \exp(-(\hat{t}-1)^2/\hat{\sigma}) \text{ where } \hat{A} \equiv At_0 \; \hat{\sigma} \equiv \frac{\sigma}{t_0^2} \tag{23}$$

To sum up, according to our model for COVID-19, the ascending behaviour of the total cases (i.e., the number of positive cases plus the cumulative number of recovered people plus the cumulative number of deaths) is given by the solution of Eq. (14) for $1/\alpha^{1/2} \leq \hat{t} \leq \hat{t}_{Max}$.

Notice that the determination of the O.D.E. (14) is of a fundamental importance if we wish also to describe the stochastic process (and the associated Fokker- Planck equation) where a white noise is added to this O.D.E. According to the literature nomenclature, we refer to the differential equation (14) as COVID-19 dynamic model[8].

5.3. Comparison between the Theoretical Predictions and Experimental Data

For Italy and Belgium one observes two distinct phases related to the dynamics of the COVID-19, which we classify as before the adoption of the lockdown measures and after some days after the adoption of the lockdown measures. The question therefore naturally arises, of whether these two types of regime are separated by a well-defined transition. We shall see that this is indeed the case. We may identify three different periods, which may be classified as follows:

1. The exponential period. As seen in Section 1, before the adoption of lockdown measures, the exponential trend is the intrinsic behaviour of the grow rate of the COVID-19.

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[8]Viral dynamics is a field of applied mathematics concerned with describing the progression of viral infections within a host organism (see, for example, [27].)

In this period the doubling time μ is a constant parameter versus time.

1. The transient period. The transient period starts after having applied the severe lockdown measures. In this period, we observe a sort of oscillations (or fluctuations) of μ versus time. In this case the time variation of μ(t) reflects the behaviour of the time effective reproduction number, R(t), defined as the number of cases generated in the current state of a population, which does not have to be the uninfected state. Fig. 6 and Fig. 7 show the behaviour of the parameter μ versus time for Italy and Belgium, respectively. The transient period ends when the last step of the exponential trend fits real data as good as the linear trend[9].

2. The bell-shaped period (or the post-transient period). In the bell-shaped period parameter μ is a (typical) function of time obtained by using Eq. (14). Several theoretical models can be used to study the post-transient period (e.g., by using Gompertz's law [28]). Here, we use two mathematical models: the solution of the differential equation (14) and the logistic model (see, for example, Ref. [16]), and we compare these two theoretical models with real data for Italy and Belgium.

(Figures 8, 9, 16) (see Appendix) compare the predictions of our model (blue lines) against the logistic model (red lines) for Italy, Belgium, and Luxembourg, respectively. Notice that the number of free parameters of these two models are exactly the same, since α and τ cannot be chosen arbitrarily. More specifically,

a) The logistic model possesses two free parameters: K and t_{0L}. Notice that parameter τ is not free since it is linked to the doubling time μ;

b) Also our model possesses two free parameters: K_N and t_0. Notice that parameter α is linked to the doubling time μ (see Eqs (14) and (18)).

(Figure 8, 9) compare the theoretical predictions, with the experimental data for Italy and Belgium updated to the 15th of May 2020. The values of the parameters τ , K_N, and t_{0L} for Eq. (14) and the parameters τ , t_{0L} and K for the logistic function are reported in the figure captions. As we can see, for both Countries Eq. (14) fits well all the real data from the initial days, while the logistic model applies only to the first data. The curves reach the plateau at the time t_{Max} given by Eq. (19). By inserting the values of the parameters, we get

$$t_{MaxIT} \simeq 80 \text{ days and } t_{MaxBE} \simeq 60 \text{ days} \qquad (24)$$

corresponding to $t_{MaxIT} \simeq 21$ April 2020 and $t_{MaxBE} \simeq 2$ May 2020 for Italy and Belgium, respectively.
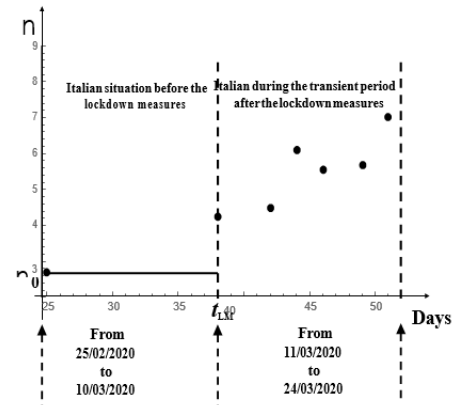
Figure 6: Italian transient period (from the 10th of March 2020 to the 24th of March 2020). During this period, the doubling time μ oscillates over time. $μ_0$ indicates the (constant) doubling time during the exponential period (for Italy $μ_0 \simeq 2.6$ days).
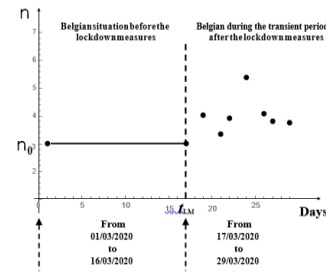
Figure 7: Belgian transient period (from the 17th of March 2020 to the 29th of March 2020). During this period, the doubling time μ oscillates over time. $μ_0$ indicates the (constant) doubling time during the exponential period (for Belgium $μ_0 \simeq 3.7$ days).

6. Modelling the COVID-19 - The Descending Phase

Here, for the descending phase is intended the phase where the number of the positive cases starts to decrease[10]. So, our model cannot be used for describing also the descending phase since N_t is the number of the total cases and, during the descending phase, N_t tends to reach the plateau. The objective of this section is to determine the trend of the curve of positive people during the descending phase. This task is accomplished by establishing

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[9]A numerical condition may be established by using the $χ^2$ test: the fittings of the two trends are considered both good if, for example, for both trends, the $χ^2$-tests get values ≥ 0.9.

[10]We define the number of positive people as the number of people tested positive for COVID- 19, hence, by excluding the number of the deceased people and the number of people who recovered.

the appropriate equations for the recovered people and the deceased people for COVID-19. During the descent phase the number of active people over time must satisfy a conservation equation. This allows determining the time-evolution for the positive people. In the sequel, we denote with r_t, d_t, and n_t the number of people released from the hospital at the time t, the total deaths, and the number of positive individuals at time t, respectively
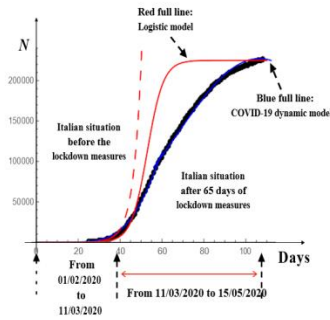

Figure 8: Situation in Italy on 15 May 2020-before, and 65 days after, the adoption of lockdown measures. The black dots correspond to experimental data. The red dotted line corresponds to the situation in Italy before the adoption of the lockdown measures. The blue and the red solid lines correspond to the theoretical predictions for Italy according to the solution of Eq. (14) and the logistic model, respectively. Solution of Eq. (14) fits well all the experimental data from the initial days (i.e., from the 1st of February 2020), while the logistic model applies only to the first days. The values of the parameters of Eq. (14) and the logistic function (10) are: $\tau_{IT} \simeq$ 3.8 days (μ_{IT} = 2.6 days), $K_N^{IT} \simeq 355250$, and $t_{0IT} \simeq 72.5$ days for Eq. (14), and $\tau_{0IT} \simeq 3.8$ days (μ_{IT} = 2.6 days), K_{IT} = 225000, t_{0LIT} = 53 days for the Logistic function.
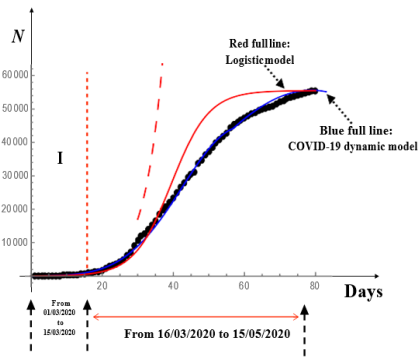

Figure 9: Situation in Belgium on 15 May 2020-before, and 60 days after, the adoption of lockdown measures. The black dots correspond to real data. The blue dotted line corresponds to the situation in Belgium before the adoption of the lockdown measures. The blue and the red solid lines correspond to the theoretical predictions for Belgium according to the solution of Eq. (14) and the logistic model, respectively. Solution of Eq. (14) fits well all the experimental data from the initial days (i.e., from the 29th of February 2020), while the

logistic model applies only to the first data. The values of the parameters of Eq. (14) and the logistic function (10) are: $\tau_{BE} \simeq$ 5.3 days (μ_{BE} = 3.7 days), $K_N^{BE} \simeq 42626$, and $t_{0BE} \simeq 53.4$ days for Eq. (14), and $\tau_{BE} \simeq 5.3$ days (μ_{BE} = 3.7 days), K_{BE} = 111000, t_{0LBE} = 39.5 days for the Logistic function, respectively. The zone I corresponds to the period before the adoption of the lockdown measures.
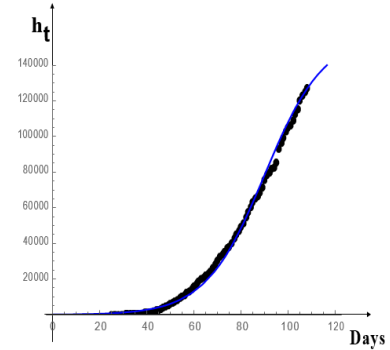
Figure 10: Italy situation. Theoretical predictions (blue line) against the experimental data (black circles) for the recovered people.
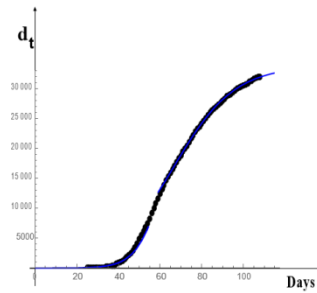
Figure 11: Italy situation. Theoretical predictions (blue line) against the experimental data (black circles) for the deceased people.

6.1 Number of the Recovered People

We start with the recovered people previously hospitalised. Let us suppose that a hospital has 50 patients in intensive therapy, corresponding to its maximum availability capacity. If the hospital is unable to heal any patient, the growth rate of healed people is necessarily equal to zero. On the other hand, if the hospital is able to heal a certain number of people, the places previously occupied by the sick people will free and other patients affected by COVID-19 will be able to be hospitalized. In the latter case, the growth rate of the healed people will rise thanks to the fact that the hospital is able to heal more and more patients. This initial phase may be modelled by introducing into the dynamic equation the term $+\gamma r_t$, with r_t indicating the number of the recovered people at the time t, previously hospitalized

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$$\frac{dr_t}{dt} = \varsigma r_t \tag{25}$$

However, it is reasonable to suppose that ζ is constant for low values of r, whereas, when r_t takes more and more large values, ζ is proportional to I_r, with I_r denoting the number of the infected people entering in the hospital (and not the total number of the infected people, which is indicated with n_t). Hence,

$$\zeta = \alpha_r - \beta_r I_r > 0 \tag{26}$$

The sign minus in Eq. (24) is due to the fact that the recovered people will continue to grow linearly until when it reaches a maximum limit i.e. until when the hospital is no longer able to accept other sick people; this causes a reduction of people who recover. The competition between these two effects diminishes the combined growth rate. Hence,

$$I_r < \frac{\alpha_r}{\beta_r} \tag{27}$$

where β_r is linked to the hospital's capacity to accept sick people. To sum up,

$$\frac{dr_t}{dt} = \alpha_r r(t) - \beta_r I_r(t - \vartheta) r(t) \quad \text{where} \quad I_r(t - \vartheta) = r(t) + D_r(t + \vartheta_1)) \tag{28}$$

r(t) is the number of the recovered people, previously hospitalised, at the time t who have been infected, in average, at the time $t - \vartheta$ and $D_r(t + \vartheta_1)$ denotes the number of deceased people at the hospital at the time $t + \vartheta_1$ who have been infected, in average, at the time $t - \vartheta$ (in general, $\vartheta_1 \neq 0$.

Clearly, the number of the recovered people, previously hospitalised, at the step n (i.e. r_n), is linked to the total number of the recovered people previously hospitalised at the step n (denoted by h_n) by the relation

$$r_n = h_n - h_{n-1} \text{ or } h_n \sum_{n=1}^{n=t/\Delta t} r_n \text{ (with } \Delta t \simeq 1 \text{ day)} \tag{29}$$

where we have set $h_0 = 0$

6.1.1. Approximated O.D.E. for the Recovered People Previously Hospitalised

We assume that all the infected people entering in the hospitals will heal. So

$$D_r(t + \vartheta_1) \approx 0 \text{ hence } I_r(t - \vartheta) \simeq r(t) \tag{30}$$

The final O.D.E. for the recovered people reads then

$$\frac{dr_t}{dt} \simeq \alpha_r \left(1 - \frac{1}{K_r} r_t\right) r_t \tag{31}$$

where K_r is the hospital's capacity, which we assume to be a time-independent parameter. It should be kept in mind that, under this approximation, the equation for the number of recovered people is in itself and independent of the equations for the other variables (i.e. for n_t and d_t)[12].

6.1.2. O.D.E. for the Total Recovered People

At the first approximation, the O.D.E. for the total recovered people R_t (i.e. the total individuals having survived the disease) is trivially obtained by considering that the rate of R_t is approximatively proportional to the number of the infected people n_t at time t i.e.[13].

$$\frac{dR_t}{dt} = \chi n_t \tag{32}$$

However, it is useful to clarify the following. In Eqs (27), h_t stands for the total number of the recovered people previously hospitalised whereas the variable R_t in Eq. (30) is the total number of the recovered people (i.e. the number of the recovered people previously hospitalised, plus the number of the asymptomatic people, plus the infected people who have been recovered without being previously hospitalised). The natural question is: "how can we count R_t and compare this variable with the real data ?". The current statistics, produced by the Ministries of Health of various Countries, concern the people released from the hospitals. A part from Luxembourg (where the entire population has been subject to the COVID-19-test), no other Countries are in a condition to provide statistics regarding the total people recovered by COVID-19. Hence, it is our opinion that the equation for R_t, is not useful since it is practically impossible to compare R_t with the experimental data.

6.2. Equation for the Deceased People

The rate of deceased people per unit time is modelled by the following dimensionless equation

$$\frac{d}{d\hat{t}} d_t = \alpha_d n_{(t-t_d)} - \beta_d n^2_{(t-t_d)} \tag{33}$$

The meaning of Eq. (31) is the following. Manifestly, the rate of deaths is proportional to the number of active people

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[11]We draw the attention of the reader that in this manuscript N_t (capital letter) represents the number of the total cases at time t, whereas n_t (small letter) refers to the number of the positive individuals at time t.

However, individuals infected by SARS-CoV-2 do not die instantly since the rate of deaths at time t is proportional to the people who were infected at an earlier time $t - t_d$ ($t_d > 0$) with td denoting the time-delay. We indicate with α_d the, time-independent, constant proportional to the increase of the deaths d_t. The second term, $-\beta_d n^2_{(t-t_d)}$, models the presence of the lockdown measures, having the effect of saturating

the rate of infected people and, consequently, of deaths. Indeed, in the absence of lockdown measures, we may approximatively write

$$\frac{d}{d\hat{t}} d_t = \alpha_r n_{(t-t_d)} \tag{34}$$

with α_r denoting a positive constant. The purpose of the lockdown measures is to decrease the number of infected people, and therefore deaths. We may assume that the effect of the lockdown measures is proportional to $n_{(t-t_d)}$ such as to dampen the linear growth of the mortality rate. In other terms, we get

$$\alpha_r \rightarrow \alpha_r - \beta_r n_{(t-t_d)} \tag{35}$$

which combined with Eq. (34) gives Eq. (33).

6.3 Equation for the Positive People

Of course, during the descent phase, the number of active people n_t satisfies a simple law of conservation: If we are in the situation where there are no longer new cases of people tested positive for COVID-19 and if we assume that the active people cannot leave their country of origin (or else, if they do, they will be rejected by the host Country), then the number of infected people cannot but decrease either because some people are deceased or because others have been recovered. In mathematical terms

$$n_t = n_0 - (h_t - h_0) - (d_t - d_0) = N_{Max} - h_t - d_t \tag{36}$$

with h_0, d_0 and t_0 denoting the values of h_t, d_t and n_t evaluated

at the time

t = t_{Max} (see Eq. (19) i.e., the time that maximises the number of the total cases), respectively. It should be noted that the conservation law (36) applies only when there are no longer new cases of people tested positive to COVID- 1914. Here, by the descending phase we mean the phase where Eq. (36) applies. To sum up, the equations describing the descending phase are

$$\frac{d}{d\hat{t}} r_t = \alpha_r r_t \left(1 - \frac{r_t}{K_r}\right) \quad \text{with } r_{t=t_{Max}} = r_0 \tag{37}$$

$$\frac{d}{d\hat{t}} d_t = \alpha_d n_{(t-t_d)} - \beta_d n^2_{(t-t_d)} \quad \text{with } d_{t=t_{Max}} = d_0$$

$$n_t = N_{Max} - h_t - d_t \quad \text{with } n_\infty = 0$$

$$h_t = \sum_{n=1}^{n=t/\Delta t} r_n \quad \text{with } \Delta t \simeq 1 \text{ day}$$

with t_{Max} given by Eq. (19). Notice that the first two equations of system (37) are valid also during the ascending-phase. Of course,

in this case, the initial conditions are $r_t=0 = 0$, $d_t=0 = 0$ and $n_t=0$. Hence, during the ascending phase the evolution equations are

$$\frac{d}{d\hat{t}} r_t = \alpha_r r_t \left(1 - \frac{r_t}{K_r}\right) \quad \text{with } r_{t=0} = 0 \tag{38}$$

$$\frac{d}{d\hat{t}} d_t = \alpha_d n_{(t-t_d)} - \beta_d n^2_{(t-t_d)} \quad \text{with } d_{t=0} = 0$$

$$n_t = N_t - h_t - d_t \quad \text{with } n_{t=0} = 0$$

$$\frac{d}{d\hat{t}} N_t = \alpha N_t \left(1 - \frac{N_t}{K_N}\right) - \left(\frac{\alpha \hat{t}^2 - 1}{\hat{t}}\right) N_t$$

$$h_t = \sum_{n=1}^{n=t/\Delta t} r_n \quad \text{with } \Delta t \simeq 1 \text{ day}$$

According to our expectations, by solving numerically system (38), with good

approximation, we get

$$d_t \propto n_{(t-t_d)} \tag{39}$$

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[12] Eq. (31) models a hospital's ability to heal people and, by no means, it must be linked to the number of people tested positive for COVID-19 or to the mortality rate caused by the SARS-CoV-2.

[13] Notice that Eq. (32) is the dynamic equation for the total recovered people adopted in the Susceptible-Infectious-Recovered-Deceased-Model (SIRD-model) [29]. The comparison between our model with the SIRD-model will be found soon in Ref. [30].

[14] So, Eq. (36) does not apply necessarily as soon as the number n_t (the number of people tested positive for COVID-19) starts to decrease. Indeed, it may happen that n_t decreases because, for example, the number of new cases of people tested positive is less than the number of the people who have recovered in the meantime. Conservation law (36) applies only from the moment where the number of new cases of people tested positive is strictly equal to zero.

Next, we find the numerical solution of systems (37)-(38) for Italy and Belgium. A similar analysis for Luxembourg is reported in Appendix.

6.4. Theoretical Predictions for the Descending Phase

In this subsection, we report the numerical solutions of Eqs (37)-(38) for Italy and Belgium. The solution for Luxembourg can be found in the Appendix. Fig. (10) and (11) concern the Italian situation. They show the numerical solution of Eqs (37)-(38) for the number of recovered people and deaths, respectively. These theoretical predictions are plotted against the experimental data reported in the (Table 1). According to the theoretical predictions, for Italy we get $t_{IT} = 12$ days. (Figure 12), illustrates the descendant-phase for Italy.
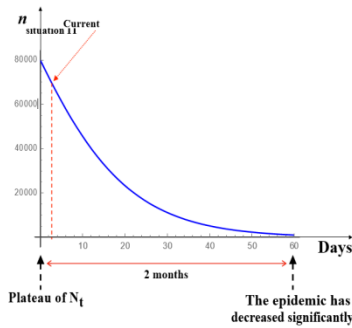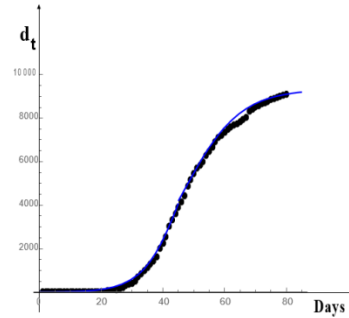

Figure 12: The descending phase for Italy. According to the theoretical predictions, after two months the lockdown measures may heavily be lightened and we can return to normal work. The estimated time-delay is $t_{Td} = 12$ days see Eq. (37).

(Figure 13, 14) refer to the Belgian situation. The figures illustrate the numerical solutions of Eqs (37)-(38) for the number of recovered people and deaths, respectively. The theoretical predictions are plotted against the experimental data reported in the (Table 3). According to the theoretical predictions, for Belgium we get $t_{BEd} = 8.8$ days. (Figure 15) shows the descendant-phase for Belgium.
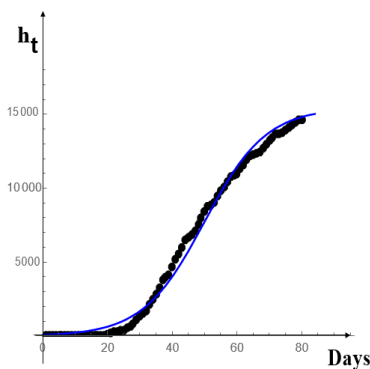
Figure 13: Belgian situation. Theoretical predictions (blue line) against the experimental data (black circles) for the recovered people.

Figure 14: *Belgian situation*. Theoretical predictions (blue line) against the experimental data (black circles) for the deceased people.
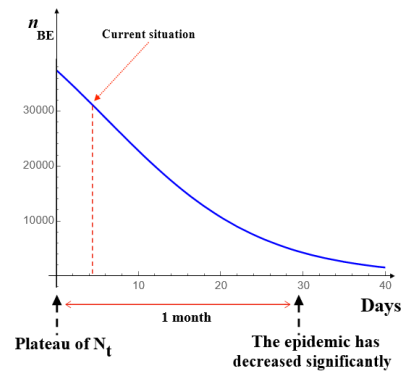
Figure 15: The descending phase for Belgium. According to the theoretical predictions, after one month the lockdown measures may heavily be lightened and we can return to normal work. The estimated time delay is $t_{BEd} = 8.8$ days see Eq. (37).

Table 3: Situation in Belgium on 15 May 2020. Columns report the number of active people (currently infected by SARS-CoV-2), the number of recovered people, and the number of deceased people.

Date	Active	Recovered	Deceased	Total cases
29-Feb	1	0	0	1
1-Mar	1	0	0	1
2-Mar	6	0	0	6
3-Mar	13	0	0	13
4-Mar	23	0	0	23
5-Mar	50	0	0	50
6-Mar	109	0	0	109
7-Mar	169	0	0	169
8-Mar	200	0	0	200
9-Mar	239	0	0	239
10-Mar	267	0	0	267
11-Mar	311	0	3	314

12-Mar	396	0	3	399
13-Mar	556	0	3	599
14-Mar	686	0	4	689
15-Mar	881	1	4	886
16-Mar	1052	1	5	1058
17-Mar	1218	20	5	1243
18-Mar	1441	31	14	1486
19-Mar	1619	155	21	1795
20-Mar	2016	204	37	2257
21-Mar	2485	263	67	2815
22-Mar	2986	340	75	3401
23-Mar	3305	350	88	3743
24-Mar	3737	410	122	4269
25-Mar	4234	547	178	4937
26-Mar	5340	675	220	6235
27-Mar	6398	858	289	7284
28-Mar	7718	1063	353	9134
29-Mar	9046	1359	431	10836
30-Mar	9859	1527	513	11899
31-Mar	10374	1696	705	12775
1-Apr	11004	2132	828	13964
2-Apr	11842	2495	1011	15348
3-Apr	12755	2872	1143	16770
4-Apr	13901	3247	1283	18431
5-Apr	14493	3751	1447	19691

6-Apr	15196	3986	1632	20814
7-Apr	16002	4157	2035	22194
8-Apr	16482	4681	2240	23403
9-Apr	17296	5164	2523	24983
10-Apr	18080	5568	3019	26667
11-Apr	18686	5986	3346	28018
12-Apr	19584	6463	3600	29647
13-Apr	19979	6707	3903	30589
14-Apr	20094	6868	4157	31119
15-Apr	22025	7107	4440	33573

last days. We found the days where the maximum number infected people by COVID-19 will be reached in Italy and Belgium by parametrising the solution of Eq. (41) with experimental data: we get, t_{MaxIT} 21 April 2020 and t_{MaxBE} 2 May 2020 for Italy and Belgium, respectively.

We also noted, empirically, that the infection process caused by SARS-CoV-2 may be divided into three qualitatively different periods; i.e., the exponential period, the transient period and the bell-shaped period (or the post-transient period). The solution of Eq. (14) allows defining more precisely these three periods. Indeed, we may classify the above periods as follows

The exponential period $\qquad$ for $\ 0 \leq \hat{t} \leq \hat{t}_{LM}$ (40)

The transient period $\qquad$ for $\hat{t}_{LM} < \hat{t} \leq \hat{t}_{flex}$

The bell-shaped period $\qquad$ for $\hat{t} > \hat{t}_{flex}$

With t_{LM} indicating the dimensionless time when the lockdown measures are applied and t_{flex} the dimensionless inflection point of the solution of Eq. (14), respectively. It is

7. Conclusions

In this work we studied the spread of SARS-CoV-2 until when the strict measures have been adopted (i.e. until 16th May 200). The dynamics of COVID-19 when the population is under less restrictive lockdown measures will be subject of future studies. Through fitting techniques previously performed, caused by viruses, including SARS-CoV-2. The solution of Eq. (14) provides the number of the total case in time. Successively, we compared the theoretical predictions, provided by the solution of Eq. (14) and by the logistic model

(see Eq. (7)), with the real data for Italy and Belgium (for Luxembourg see Appendix). We saw that the solution of Eq. (14) is in good agreement with the experimental data since the beginning of the appearance of the COVID-19; this is not the case for the logistic model which applies only to the few

easily checked that, for large values of K_N, the value of t^f lex satisfies, approximatively, the equation

$$\alpha \hat{t}^3{}_{flex} - 2\alpha \hat{t}^2{}_{flex} + (\alpha - 3)\hat{t}_{flex} + 2 \simeq 0 \quad \text{with} \quad \hat{t}_{flex} \equiv \frac{t_{flex}}{t_0} \quad (41)$$

Hence, according to Eq. (41), the transient period ended on 31 March 2020 for Italy and on 7 April 2020 for Belgium, respectively. The second part of the work is devoted to modelling the descending phase, i.e. the decrease of the number of people tested positive for COVID-19. Also in this case, we proposed a new set of dynamic differential equations that we solved numerically. The solution of Eqs (37) (and Eq. (38)) provided valuable information such as the duration of the COVID-19 epidemic in a given Country and therefore when it will be possible to return to normal life.

8. Acknowledgments

Embassy in Belgium for his suggestions and encouragement.

9. Appendix: Comparison between the Theoretical Predictions of Eq. (14) and Experimental Data for Luxembourg

We have stressed the main difference between the closed systems and the open systems. Luxembourg, due to the particularly severe lockdown measures adopted by the government, may be considered, with good approximation, as a closed system (628108 inhabitants, most of them concentrated in only one town). Indeed, right from the start, the city of Luxembourg was literally closed and citizens were unable to enter and leave the city freely (people who had to enter in the city for working reasons were obliged to undergo each time the test that, of course, had to result negative).

Italy, on the other hand may be considered, with a god approximation, as an open system (60317116 inhabitants dislocated in all the Country). In Italy, especially during the initial phase, the citizens of northern Italy moved freely to the south of Italy, by train, by plans or by car. Only in a second time the Italian government decided to introduce much more restrictive measures concerning the movement of citizens from one region to another.

For the reason mentioned above, it is our opinion that it is very interesting to analyse these two Countries, Luxembourg and Italy, which are so different with each other. In this Appendix we report the comparison between the theoretical predictions of the COVID-19 model (14) and the real data for Luxembourg update to 15 May 2020 (see (Figure 16)). In the columns of (table 5) we can find the number of active people (currently infected by SARS-CoV-2), the number of recovered people, and the number of deceased people, respectively. The experimental data have been found in the databases [31, 32]. Luxembourg reached its peak on 12 April 2020.

Table 4: Situation in Belgium on 15 May 2020. Columns report the number of active people (currently infected by SARS-CoV-2), the number of recovered people, and the number of deceased people.

Date	Active	Recovered	Deceased	Total cases
16-Apr	22390	7562	4857	34809
17-Apr	23014	7961	5163	36138
18-Apr	23346	8384	5453	37183
19-Apr	24056	8757	5683	38496
20-Apr	25260	8895	5828	39983
21-Apr	25296	9002	5998	40956
22-Apr	26194	9433	6262	41889
23-Apr	26507	9800	6490	42797
24-Apr	27492	10122	6679	44293
25-Apr	27991	10417	6917	45325
26-Apr	28255	10785	7094	46134
27-Apr	28602	10878	7207	46687
28-Apr	29060	10943	7331	47334
29-Apr	29075	11283	7501	47859
30-Apr	29349	11576	7594	48519
1-May	29437	11892	7703	49032
2-May	29541	12211	7765	49517
3-May	29753	12309	7844	49906
4-May	29965	12378	7924	50267
5-May	30052	12441	8016	50509

6-May	29711	12731	8339	50781
7-May	30025	12980	8415	51420
8-May	30289	13201	8521	52011
9-May	30604	13411	8581	52596
10-May	30783	13642	8656	53081
11-May	31045	13697	8707	53449
12-May	31286	13732	8761	53779
13-May	31201	13937	8843	53981
14-May	31274	14111	8903	54288
15-May	31384	14301	8959	54644
16-May	31524	14460	9005	54986
17-May	31598	14630	9052	55280
18-May	31822	14657	9080	55559
19-May	31996	14687	9108	55791
20-May	31986	14847	9150	55983
21-May	32061	14988	9186	56235
22-May	32176	15123	9212	56810
23-May	32418	15155	9237	56810
24-May	32540	15272	9280	57092
25-May	32733	15297	9312	57342
26-May	32801	15320	9334	57455
27-May	32763	15465	9364	57592
28-May	32889	15572	9388	57849
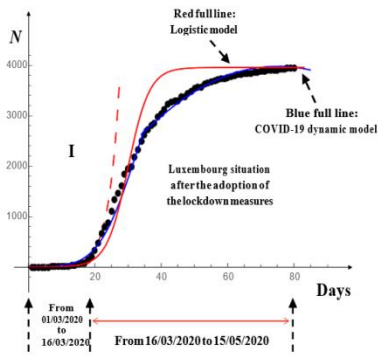
Figure 16: Situation in Luxembourg on 15 May 2020. The black dots correspond to real data. The red dotted line corresponds to the situation in Luxembourg before the adoption of the lockdown measures. The blue and the red solid lines correspond to the theoretical predictions for Luxembourg according to the solution of Eq. (14) and the logistic model, respectively. Solution of Eq. (14) fits well all the experimental data from the initial days (i.e., from the 29th of February 2020), while the logistic model applies only to the first data. The values of the parameters of Eq. (14) and the logistic function (10) are: τ_{LUX} c3.2 days (μ_{LUX}= 2.2 days), $K_N^{LUX} \simeq 6880$, and $t_{0LUX} \simeq 40$ days for Eq. (14), and τ_{LUX} 3.2 days ($\mu_{LUX} = 2.2$ days), $K_{LUX} = 3950$, $t_{0LLUX} = 30$ days for the Logistic function, respectively. The zone I corresponds to the period before the adoption of the lockdown measures.

Table 5: Situation in Luxembourg on 15 May 2020. Columns provide the number of active people (currently infected by SARS-CoV-2), the number of recovered people, and the number of deceased people.

Date	Active	Recovered	Deceased	Total cases
29-Feb	1	0	0	1
1-Mar	1	0	0	1
2-Mar	1	0	0	1
3-Mar	1	0	0	1
4-Mar	1	0	0	1
5-Mar	2	0	0	2
6-Mar	3	0	0	4
7-Mar	4	0	0	4
8-Mar	5	0	0	5
9-Mar	5	0	0	5
10-Mar	7	0	0	7
11-Mar	7	0	0	7
12-Mar	26	0	0	26
13-Mar	33	0	1	34
14-Mar	50	0	1	51
15-Mar	76	0	1	77
16-Mar	80	0	1	81
17-Mar	139	0	1	140
18-Mar	201	0	2	203

Date	Active	Recovered	Deceased	Total cases
19-Mar	325	6	4	335
20-Mar	474	6	4	484
21-Mar	656	6	8	670
22-Mar	780	10	8	798
23-Mary	857	10	8	875
24-Mar	1081	10	8	1099
25-Mar	1315	10	8	1333
26-Mar	1434	10	9	1453
27-Maryy	1550	40	15	1605
28-Mar	1773	40	18	1831
29-Mar	1889	40	21	1950
30-Mar	1926	40	22	1988
31-Mar	2115	40	23	2178
01-Apryy	2250	40	29	2319
2-Apr	2417	40	30	2487
3-Apr	2081	500	31	2612
4-Apr	2198	500	31	2729
5-Apr	2268	500	36	2804
6-Apr	2302	500	41	2843
7-Apr	2426	500	44	2970
8-Apr	2488	500	46	3034
9-Apr	2563	500	52	3115
10-Apr	2669	500	54	3223
11-Apr	2708	500	62	3270
12-Apr	2715	500	66	3281
13-Apr	2725	500	67	3292
14-Apr	2740	500	67	3307
15-Apr	2778	520	69	3373

Y Attention on the 23rd the figures instead of being given at 9 am are given at 5 pm.

YY Including 1 evacuated from France.

Table 6: Situation in Luxembourg on 15 May 2020. Columns provide the number of active people (currently infected by SARS-CoV-2), the number of recovered people, and the number of deceased people.

Date	Active	Recovered	Deceased	Total cases
16-Apr	1008	2368	68	3444
17-Apr	919	2489	72	3480
18-Apr	858	2607	72	3537
19-Apr	799	2678	73	3550
20-Apr	759	2724	75	3558
21-Apr	735	2805	78	3618
22-Apr	689	2885	80	3654
23-Apr	619	2963	83	3665

24-Apr	582	3028	85	3695
25-Apr	539	3087	85	3711
26-Apr	532	3103	88	3723
27-Apr	520	3121	88	3729
28-Apr	531	3121	89	3741
29-Apr	546	3134	89	3769
30-Apr	481	3213	90	3784
1-May	497	3213	92	3802
2-May	402	3318	92	3812
3-May	349	3379	96	3824
4-May	327	3405	96	3828
5-May	332	3412	96	3840
6-May	301	3452	98	3851
7-May	254	3505	100	3859
8-May	245	3526	100	3871
9-May	226	3550	101	3877
10-May	199	3586	101	3886
11-May	185	3602	101	3888
12-May	182	3610	102	3894
13-May	172	3629	103	3904
14-May	147	3665	103	3915
15-May	137	3682	104	3923
16-May	127	3699	104	3930
17-May	136	3702	107	3945
18-May	125	3715	107	3947
19-May	131	3718	109	3958
20-May	134	3728	109	3971
21-May	130	3741	109	3980
22-May	124	3748	109	3981
23-May	123	3758	109	3990
24-May	115	3767	110	3992
25-May	102	3781	110	3993
26-May	102	3783	110	3995
27-May	100	3791	110	4001
28-May	95	3803	110	4008

9.1. The Descending Phase for Luxembourg

(Figures 17, 18) refer to the Luxembourg situation. The figures illustrate the numerical solutions of Eqs (37) -(38) for the number of recovered people and deaths, respectively. The theoretical predictions are plotted against the experimental data, which can be found in the (Table 5 Figure 19) shows the descending phase for Luxembourg.
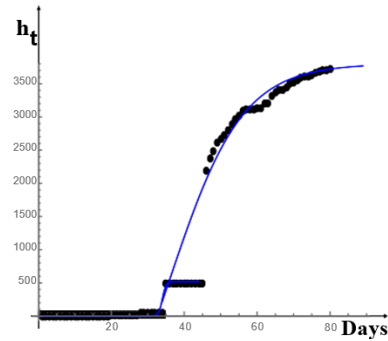

Figure 17: Luxembourg situation. Theoretical predictions (blue line) against the experimental data (black circles) for the recovered people.
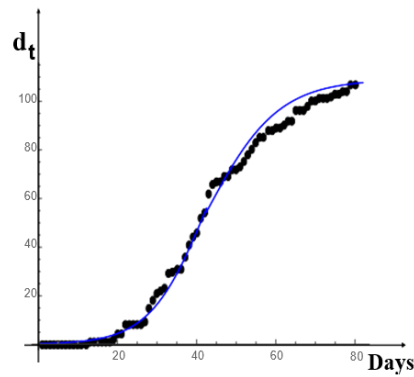

Figure 18: Luxembourg situation. Theoretical predictions (blue line) against the experimental data (black circles) for the deceased people.
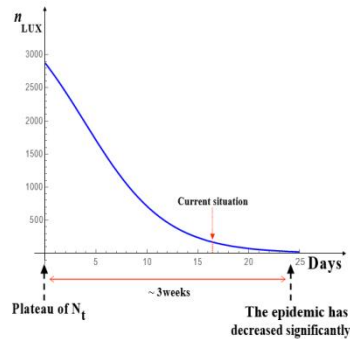

Figure 19: *The descending phase for Luxembourg*. According to the theoretical predictions, after one month the lockdown measures may heavily be lightened and we can return to normal work. The estimated t imedelay is $t_{LUXd} = 15$ days see Eq. (37).

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